Modeling Engineering Applications Using Simulink

Chapter 2

Modeling Tank level control Using Simulink

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Aim of Chapter 2

The aim of this laboratory assignment is to model a water tank and simulate it in closed-loop.

Learning objectives

- • Building models in SIMULINK.
- • Linearization of non-linear plants.
- • PID-controllers.
- • Anti-wind-up schemes

Introduction

Consider an open water tank with cross-sectional area *A, see figure1* *Water is pumped into* the tank at the top at rate of flow of q_{in} *cubicmetres per second. Water is flowing out of the* tank through a hole in the bottom of the tank of area *a.*

The rate of flow of water through the hole is according to the Bernoulli equation given by

$$q_{out} = a\sqrt{2gh}\,, \qquad (1.)$$

where h is level of tank and g is the acceleration of gravity. Conservation of mass yields the equation

Details of Water Control System

$$A\frac{dh}{dt} = q_{in} - q_{\text{out}} = q_{in} - a\sqrt{2gh} \quad . \qquad (2.)$$

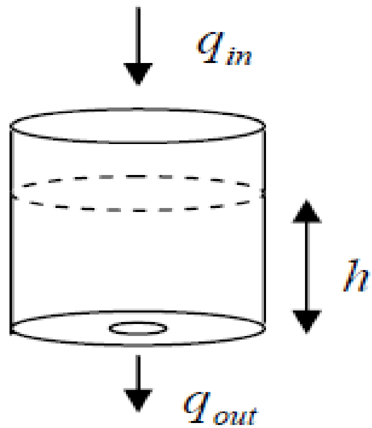


Figure 1.

The parameters in equation (2) are

$$\begin{aligned} A &= 2.3\cdot 10^{-3}\ \text{m}^2 \\ a &= 7.1\cdot 10^{-6}\ \text{m}^2 \\ g &= 9.82\ \text{m/s}^2 \end{aligned} \qquad (3.)$$

Exercise 1

The objective of this exercise is to simulate the tank in Simulink .
Equation (2) is first rewritten as an integral equation

$$h(t) = h(0) + \int_0^t \frac{1}{A}\left(q_{in}(t') - q_{\text{out}}(t')\right)dt' = h(0) + \int_0^t \frac{1}{A}\left(q_{in}(t') - a\sqrt{2gh(t')}\right)dt' \qquad (4.)$$

The Simulink model corresponding to equation (4) can be seen if figure 2.

Follow these steps in order to build the model in figure 2:

1. Start Matlab by clicking on the Matlab icon on the desktop.

2. Change the current directory to your preferred working directory by typing that directory name into the current-directory-box in Matlab.

3. Start the Simulink Library Browser by clicking on the Simulink icon in the toolbar of the Matlab window.

Exercise 1

4. Open a new model by selecting ‘New’ from the ‘File’ menu in the Simulink Library Browser.
5. Blocks from the Simulink Library Browser can be dragged into the model window. In this example a ‘Step’ block from the ‘Sources’ folder gives the step input signal. The output is presented om a ‘Scope’ block from the ‘Sinks’ folder. The gain block is found in the ‘Commonly Used Blocks’ folder while the integrator block is found in the ‘Continuous’ folder. The ‘Fcn’ block resides in the ‘User-Defined Blocks’. Variables like ‘A’ or ‘a’ defined in Matlab may be used in the Simulink blocks, as can be seen in figure 2..
6. The blocks may be connected by wires by first highlighting one block and then clicking on another while pressing the control key. A model of the water tank is shown in figure 1.
7. Simulate how the water level responds to a step in the flow from 0 to 10−5 m3/s .
8. The simulation is started by pressing the play button in the tool panel. Set the simulation time to 400 s in the box right of the stop button.
9. What is the steady-state value? Verify the steady-state value obtained from the simulation by comparing it to the theoretical value.

Exercise 1

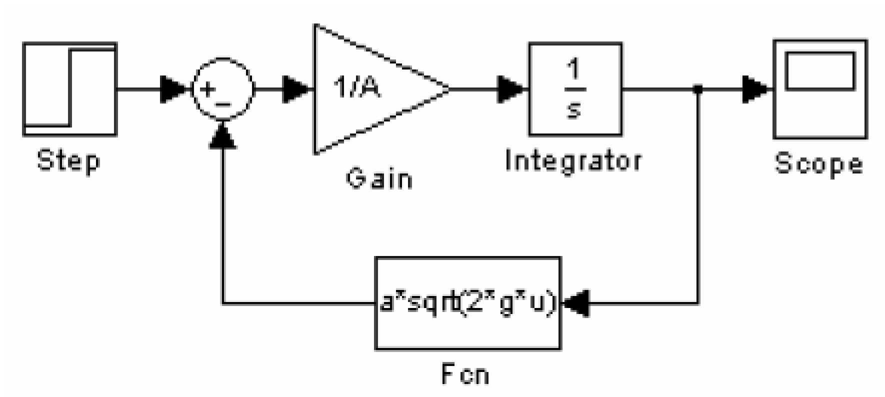


Figure 2. Simulink model of water tank.

Linearization

The model of the water tank is non-linear. The non-linear model can be approximated by a linear model for small deviations around an operating point.

Exercise 2

Linearize the water tank model. Compare the original model and the linearized model by doing simulations. Consider a step increase in the flow into the tank of 10−6 m3/s from an equilibrium operating point corresponding to a water level of 0.10 m. Vary the size of the step of the flow into the tank.

Closed-loop

The actuator is a pump which pumps water into the water tank. The relationship between the rate of flow in q and the controller output u is

$$q_{in} = ku \,, \qquad (5.)$$

where k = 3.9 · 10^{-6} m3 /(s ·V) . The pump saturates at an input voltage of 10 V. The pump cannot draw water from the tank, so there is also saturation at zero input voltage. The water level is measured by a sensor whose sensitivity is 40 V/m. A model of the water tank in closed-loop together with the actuator and the controller can be seen in figure 2.

Exercise 2

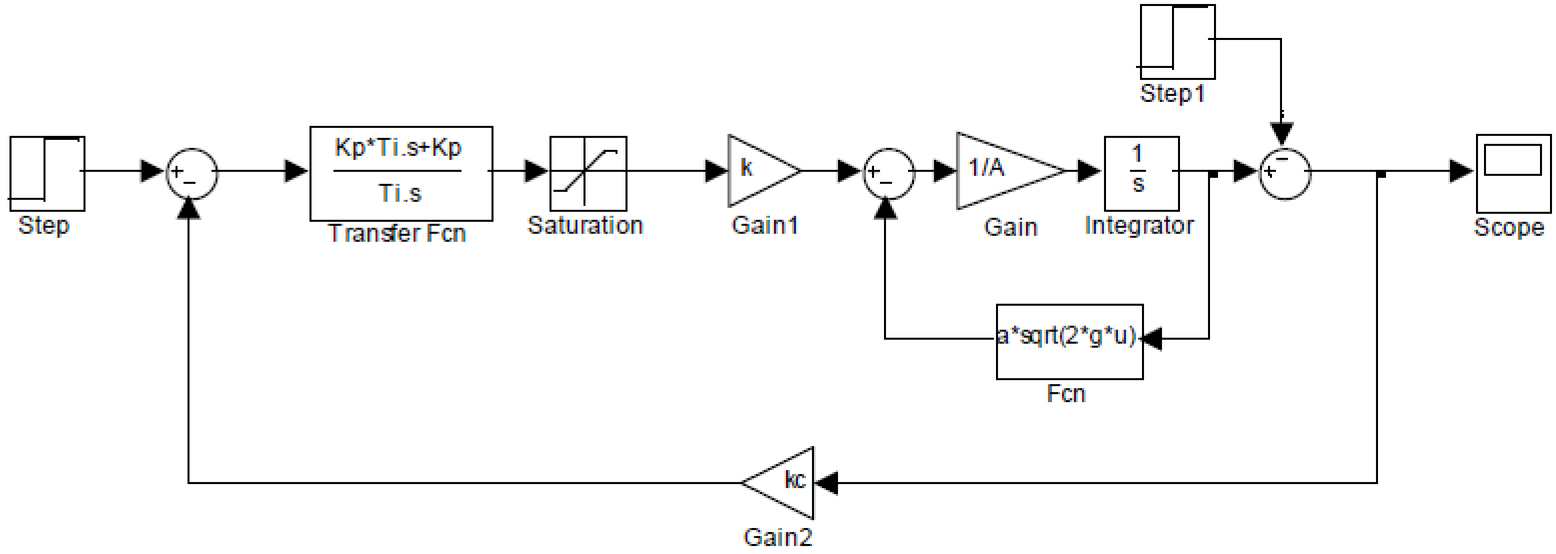


Figure 2. Water tank in closed loop.

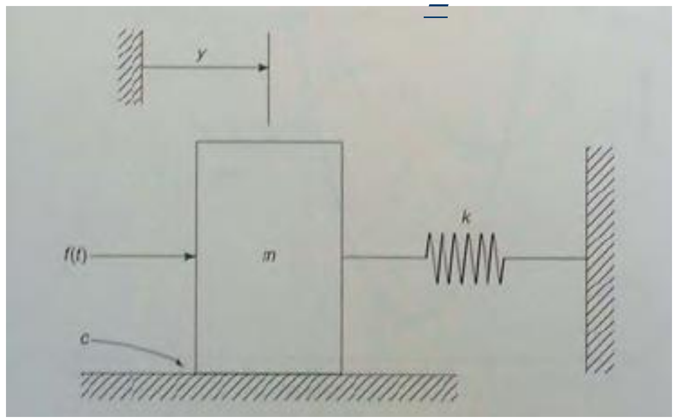
Exercise 3

Synthesize a PI-controller using the pole-placement technique. Locate the poles at $s = -0.020 \pm 0.015\, j$ rad/s . Simulate the water tank in closed-loop with the controller.

Investigate the responses to step references and disturbances. Calculate the proportional gain and reset time. These values will be used in the experimental laboratory (part 2).

EXO_4. MATLAB Application in **Mass_Spring_Damping.mdl**

- **EXAMPLE 4.** A mass connected to spring with viscous friction acting between the mass and the surface



The equation describes the motion : $$m\ddot{y} + c\dot{y} + ky = f(t)$$

The equation can be put into Cauchy form by letting $x_1 = y$ and $x_2 = \dot{y}$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{1}{m}f(t) - \frac{k}{m}x_1 - \frac{c}{m}x_2$$

These two equations can be writing as one matrix equation : $$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{k}{m} & -\frac{c}{m}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m}\end{bmatrix}f(t)$$

EXO_4. MATLAB Application in **Mass_Spring_Damping.mdl**

Is *compact form this is:* $\dot{x} = A.x + B.f(t)$

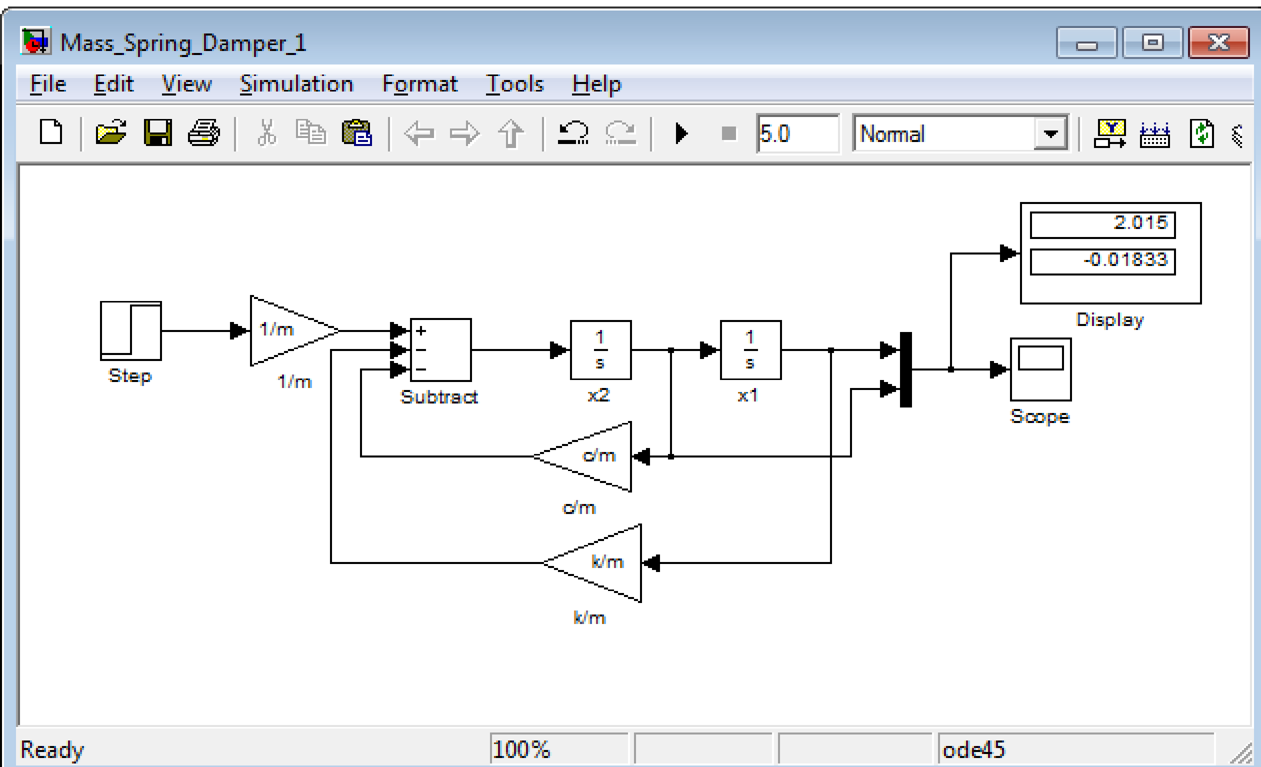
Where $A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$ <u>*and*</u> $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

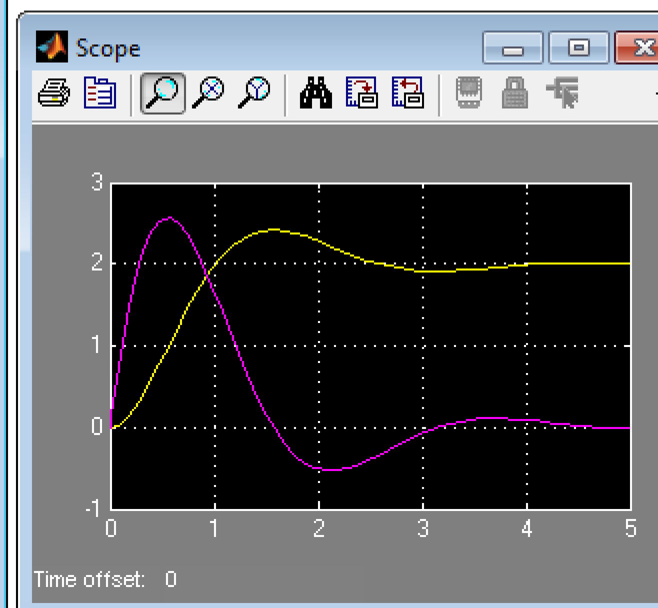
In this example m = 1, c = 2, k = 5, and the applied force f is a constant equal to 10.

Program m.File

```
m=1;c=2;k=5;f=10;
A=[0 1;-k/m -c/m];
B=[0;1/m];
Mass_Spring_Damper_1
Mass_Spring_Damper_2
Mass_Spring_Damper_3
```

Solution_1: *Building models in SIMULINK* (Mass_Spring_Damping.mdl)





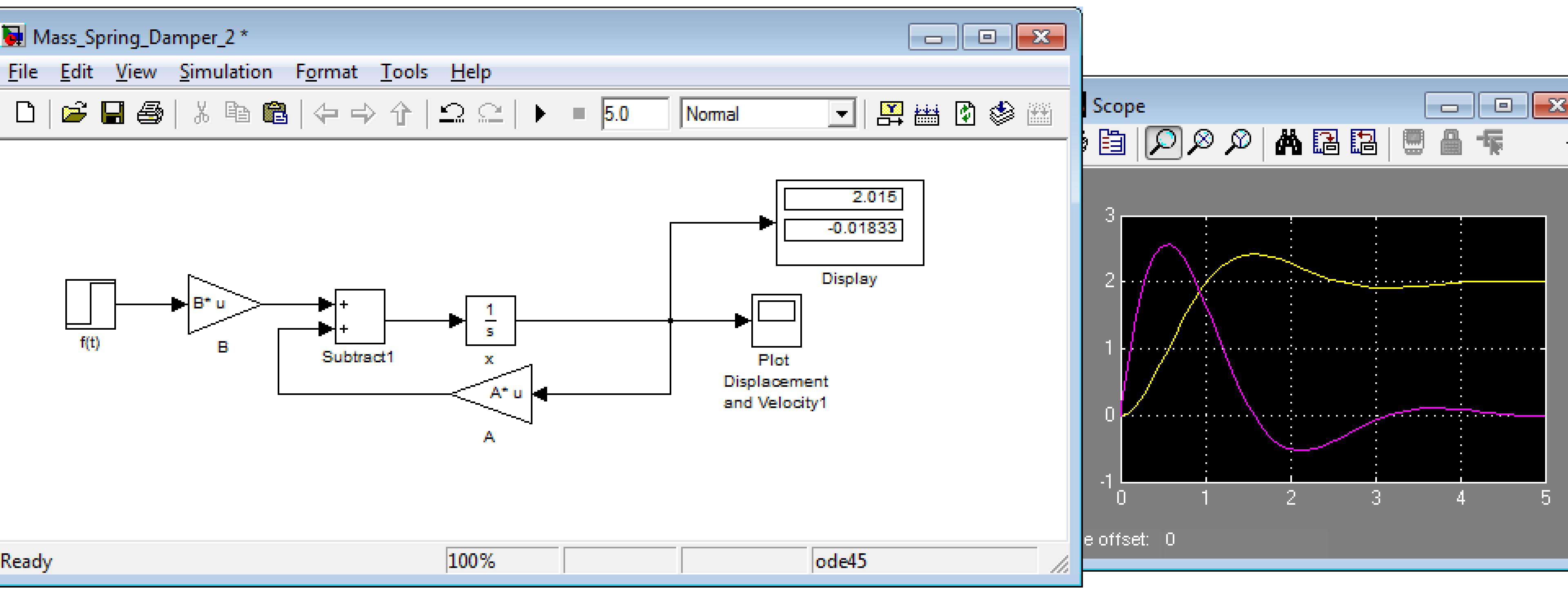
EXO_4. MATLAB Application in **Mass_Spring_Damping.mdl**

Is *compact form this is:* $\dot{x} = A.x + B.f(t)$

Where $A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$ <u>*and*</u> $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

In this example m = 1, c = 2, k = 5, and the applied force f is a constant equal to 10.

Solution_2: ***Building models in SIMULINK*** **(Mass_Spring_Damping.mdl)**



EXO_4. MATLAB Application in **Mass_Spring_Damping.mdl**

Is *compact form this is:* $\dot{x} = A.x + B.f(t)$

Where $A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$ *and* $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

In this example m = 1, c = 2, k = 5, and the applied force f is a constant equal to 10.

Solution_3: ***Building models in SIMULINK*** **(Mass_Spring_Damping.mdl)**